MS115 Mathematics for Enterprise Computing

First in-class test - Answer ALL questions

Name	Student Id

1. In each part of this question, circle the number of **one** correct answer.

   Marking scheme for Question 1: +3 for each correct answer; −1 for each incorrect answer; 0 marks for no answer or an unclear answer.

   (a) Two sets A and B are equal when

   (*i*) $\cancel{A \cap B \subseteq A}$;

   (*ii*) $A \subseteq B$ and $B \subseteq A$;

   (*iii*) $\cancel{A \cap B = \emptyset}$;

   (*iv*) $\cancel{A \cap B \neq \emptyset}$.

   (b) Sets A, B and C are pairwise disjoint if

   (*i*) $\cancel{A \cap B = C}$;

   (*ii*) $\cancel{A \cap B \cap C = \emptyset}$;

   (*iii*) $\cancel{A \cap B \cap C = (A \cap B) \cap C}$;

   (*iv*) $A \cap B = \emptyset$, $B \cap C = \emptyset$ and $A \cap C = \emptyset$.

   (c) A *function* from a set A to a set B is a relation on A and B that

   (*i*) ~~relates some element of A to some element of B~~;

   (*ii*) ~~relates exactly one element of A to some element of B~~;

   (*iii*) relates each element of A to exactly one element of B;

   (*iv*) ~~relates every element of A to more than one element of B~~.

   (d) A *function* from a set A to a set B is invertible if

   (*i*) $\cancel{|A| \geq |B|}$;

   (*ii*) ~~no element of A is related to more than one element of B~~;

   (*iii*) every element of B is the image of exactly one element of A;

   (*iv*) ~~every element of B is the image of at least one element of A~~.

   (e) The relation $\leq$ on the set of integers $\mathbb{Z}$ is

   (*i*) ~~reflexive, symmetric and transitive~~;

   (*ii*) ~~reflexive and symmetric, but not transitive~~;

   (*iii*) reflexive and transitive, but not symmetric;

   (*iv*) ~~symmetric and transitive, but not reflexive~~.

2. Compute the truth table of the following compound proposition:

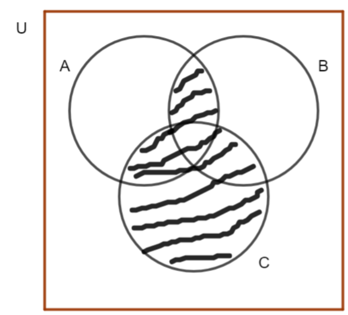
$$(P \Rightarrow Q) \vee (P \vee Q).$$

[10 marks]

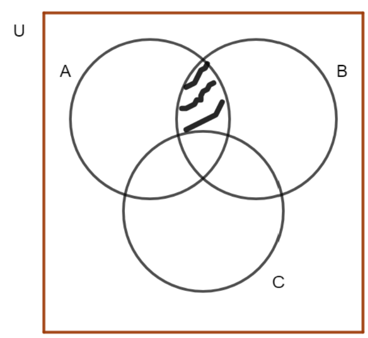
P	Q	$P \Rightarrow Q$	$P \vee Q$	$(P \Rightarrow Q) \vee (P \vee Q)$
T	T	T	T	T
T	F	F	T	T
F	T	T	T	T
F	F	T	F	T

3. In each case, shade the relevant region of the given Venn diagram:

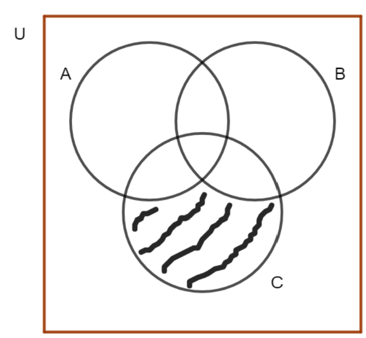
(i) $(A \cap B) \cup C$



(ii) $(A \cap B) \cap \overline{C}$



(iii) $(\overline{A} \cap \overline{B}) \cap C$



[6 marks]

4. Consider two sets A and B within a universal set U. Determine the number of elements in $A \cap B$ given the following set cardinalities:

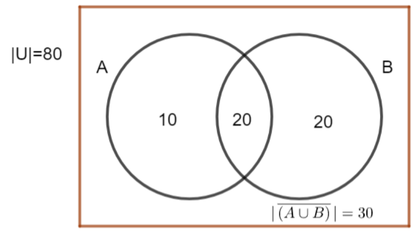
$$|\,U\,| = 80, \qquad |\,\overline{A}\,| = 50, \qquad |\,\overline{B}\,| = 40, \qquad |\,\overline{(A \cup B)}\,| = 30.$$

(You may find it helpful to sketch a Venn diagram) [6 marks]

As $|\,\overline{A}\,| = 50$ and $|\,\overline{(A \cup B)}\,| = 30$, we can conclude that $|\,\overline{A} \cap B\,| = 20$.

As $|\,\overline{B}\,| = 40$ and $|\,\overline{(A \cup B)}\,| = 30$, we can conclude that $|\,A \cap \overline{B}\,| = 10$.

Thus, as $|\,U\,| = 80$, $|\,\overline{(A \cup B)}\,| = 30$, $|\,\overline{A} \cap B\,| = 20$ and $|\,A \cap \overline{B}\,| = 10$, it follows that $|\,A \cap B\,| = 20$.



5. For $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, consider the equivalence relation R on A defined as follows:

$$xRy \text{ exactly when } y - x = 3k \text{ for some } k \in \mathbb{Z}.$$

Write down the distinct equivalence classes that form a partition of A.

(Note: For $x \in A$, the equivalence class of x is the set $E_x = \{y \in A \mid yRx\}$).

[7 marks]

We have that $1R4$, $1R7$ and $1R10$. Hence $E_1 = \{1, 4, 7, 10\}$.

As $2R5$ and $2R8$, we have that $E_2 = \{2, 5, 8\}$.

As $3R6$ and $3R9$, we have that $E_3 = \{3, 6, 9\}$.

The equivalence classes that partition A are $\{1, 4, 7, 10\}$, $\{2, 5, 8\}$ and $\{3, 6, 9\}$.

6. Consider the function

$$f(x) = \frac{x+1}{2x-2}$$

(i) Let its domain be the largest possible subset of $\mathbb{R}$. Describe this set.
The function f is defined for all $x \neq 1$.
Hence, $\mathbb{R} - \{1\}$ is the largest possible domain of f.

(ii) Show that $\frac{1}{2}$ is not an element of the range of f.
The equation $\frac{x+1}{2x-2} = a$ does not have a solution for $a = \frac{1}{2}$, as

$$\frac{x+1}{2x-2} = \frac{1}{2} \Rightarrow 2(x+1) = 2x - 2 \Rightarrow 2 = -2, \text{ a contradiction.}$$

[6 marks]