1

MS115

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CA - exam breakdown : 25% - 75%
CA - 2 in-class tests

Course Outline

- Logic
- Sets
- Relations and functions
- Combinatorics (counting)
- Statistics

Logic

A proposition is a statement that has a truth value; it is either true (T) or false (F)

Examples:
- The world is flat
- I am wearing a red dress
- 3 is a prime number
  (only positive ~~whole~~ number divisors are 1 & 3)

Non-examples:
- What's the time?
- Ireland will win the 2019 Rugby World Cup.
  (prediction)
- This statement is not true.
  Can we assign a truth value to this?

<u>Notation</u>: We'll use capital letters to represent our propositions:

eg. P: The next Eurovision will be in Israel

Q: It is raining in DCU now.

R: The sky is blue.

We'll use logical operators to connect our simple propositions (P, Q and R) to form (generally more complicated) <u>compound propositions</u> (involving a combination of P, Q and R).

- We have the negation operator, <u>not</u>.

This is defined so that
not P is T when P is F
and not P is F when P is T.

eg. not P: The next eurovision will not be in Israel.

We can show this definition using a truth table.

P	not P
T	F
F	T

③

- We'll also wish to combine propositions using the AND operator, denoted $\wedge$.

We define $P \wedge Q$ (P and Q) to be true is P is true and Q is true.

Spelling this out with a truth table:

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

(AND is sometimes called "conjunction")

- We also have the OR operator, denoted $\vee$.

We define $P \vee Q$ (P or Q) to be true if <u>either</u> P is true or Q is true.

More specifically, we have truth table

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

This shows that our OR operator is not the "exclusive OR" operator.

The OR operator is sometimes called "disjunction".

- We can build more complicated compound propositions using a combination of these operators. We use brackets to clarify the order in which operations are evaluated. Once our truth tables consider every combination of truth values, everything is clear.

For example, let's look at the truth table of ((not P) and Q) or R

P	Q	R	(not P)	(not P) and Q	((not P) and Q) or R
T	T	T	F	F	T
T	T	F	F	F	F
T	F	T	F	F	T
T	F	F	F	F	F
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	F	T
F	F	F	T	F	F

"Fun" puzzle:

We've 3 people, P1, P2 & P3, arranged in a row.
P_1 is at the front, etc.

$\longleftarrow$

P_1 P_2 P_3

P_1 can see noone,
P_2 can see P_1 &
P_3 can see everyone else.

Someone puts hats on their heads & says the true statement:
there is ≥ 1 red hat,
where the hats are red or blue.

P_3 says "I know my hat colour".
What are the colours for P_1, P_2, P_3?
P_1 blue, P_2 blue & P_3 red

Same game (new hats & ≥ 1 red)

P_3 says "I don't know my colour".
P_2 says "I don't know my colour".

What are the colours? ?

What colour is P[redacted]'s hat?

P[redacted] red

Let's use a truth table:

6

A : P_1 has a red hat
B : P_2 " " " "
C : P_3 " " " "

Given $A \vee B \vee C$ is T

A	B	C	$(A \vee B) \vee C$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	T $\rightarrow$ X
F	F	F	F

Problem 1 : P_3 knows their colour

We want to also use the conditional operator to construct arguments.

Notation : $P \Rightarrow Q$ or $P \rightarrow Q$

Meaning : if P is true, then Q is true

Definition via truth table :

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

"vacuously"

P : It is raining

Q : I drive to DCU

If P is true and Q is true, then $P \Rightarrow Q$ true

If P is true and Q is false, etc.

- Two propositions are <u>logically equivalent</u> if they have identical columns in a truth table.

We write $P \equiv Q$ to denote this.

For example, we can show

$$P \Rightarrow Q \equiv (\text{not } Q) \Rightarrow (\text{not } P)$$

P	Q	not P	not Q	$P \Rightarrow Q$	(not Q) $\Rightarrow$ (not P)
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

Here, (not Q) $\Rightarrow$ (not P) is called the <u>contrapositive</u> of $P \Rightarrow Q$