CHAPTER 2 : SETS

* GENERALITIES

"DEFINITION": For us, a set is a collection of objects, called elements.

Notation: we use curly brackets and commas between elements.

ex: $S = \{1, 2, 4\}$.

REMARKS:

- There is no order between the elements of a set:

  e.g. $\{1,2,4\} = \{2,4,1\} = \{4,1,2\}$.

- We allow ourselves to consider sets with anything inside:

  $\{\text{ME, THE UNIVERSE}\}$; $\{\{1,2\}, \{2\}\}$

  are sets.

* Definition using propositions: we will define set of the shape $S = \{x \in E \text{ s.t. } P(x)\}$,

where E is a bigger set and $P(x)$ a proposition whose truth value depends on x.

ex: $S = \{ n \in \mathbb{N} \text{ s.t. } \underbrace{n \text{ is even}}_{P(n)} \}$.

the set of natural numbers

Notation: we write "$x \in S$" for
"x is an element of S".
(and we've already used it in this notes, my apologies).

* **VENN DIAGRAMS:** we will use VENN diagrams along the notes to illustrate the "relations" between various sets.

* **SETS OF NUMBERS:**

$\mathbb{N} = \{1, 2, 3, \ldots\}$ is the set of natural numbers

$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3\}$ is the set of integers

$\mathbb{Q} = \left\{ \frac{p}{q} \;\middle|\; p \text{ is an integer and } q \text{ is a natural number} \right\}$

is the set of rational numbers.

$\mathbb{R} = \{ \underline{\text{all}} \text{ decimal numbers} \}$ is the set of

real numbers.

<u>ex</u>: $\frac{1}{3} \in \mathbb{Q}$, π and $\sqrt{2} \in \mathbb{R}$

but π and $\sqrt{2} \notin \mathbb{Q}$

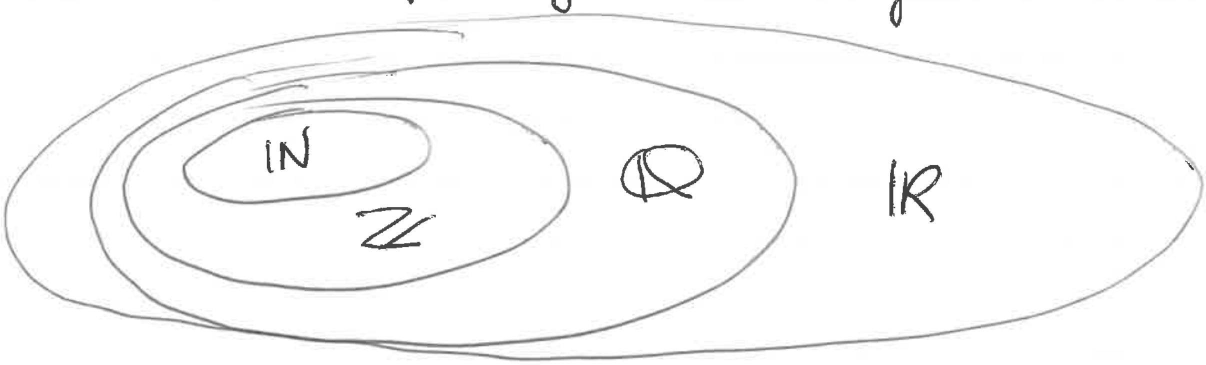
"are not element of"

- We have

$\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$,

i.e. $\mathbb{N}$ is contained in $\mathbb{Z}$, etc.

we have the following VENN diagram:



* INCLUSION AND SUBSETS:

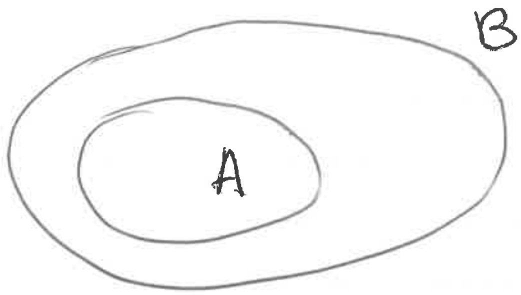
More generally, we write

$A \subseteq B$ and we say that

A is INCLUDED in B or

A is a SUBSET of B if

$$x \in A \Longrightarrow x \in B$$



* THE EMPTY SET

The empty set is a set with nothing inside!

It is denoted by $\{\ \}$ or $\emptyset$.

REMARK: For every set A we have

$\emptyset \subset A$, since every element in $\emptyset$

are also in A (this is a VACUOUS TRUTH).

* EQUALITY OF SETS:

Two sets are equal when they have the same elements.

We have: $A = B$

$\iff \left(x \in A \iff x \in B \right)$

$\iff \quad A \subseteq B$ AND $B \subseteq A$.

example:

$\{ n \mid n^2$ is an odd integer$\}$

$= \{ n \mid n$ is an odd integer$\}$.

(see paragraph on proving implications).

* WHAT ARE ALL THE POSSIBLE SUBSETS OF A SET?

example: $A = \{1, 2, 3\}$.

Let us list the subsets of A:

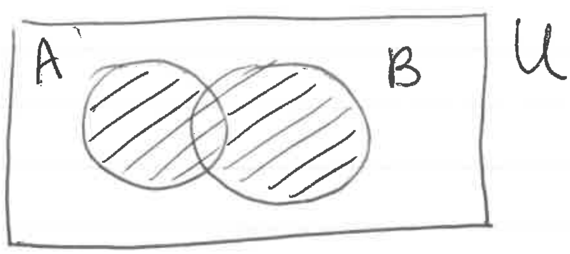
$\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, \{1,2,3\}$

REMARK This example shows that a set with 3 elements has $2^3 = 8$ subsets. We will see that in general, a set with n elements has 2^n subsets.

* SET OPERATIONS

- The union of two sets A, B is the set

$$A \cup B = \{x \in U \mid x \in A \text{ OR } x \in B\}$$

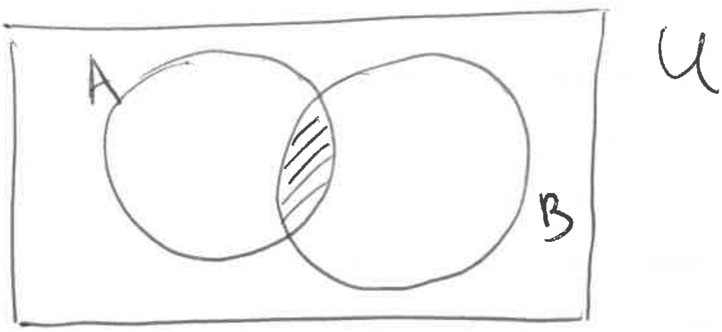


REMARK: $\mathcal{U}$ is a set, "the Universe" that is supposed to be' such that

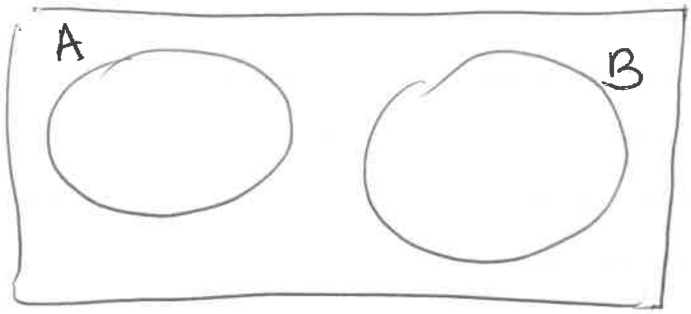
$$A \subseteq \mathcal{U} \quad \text{and} \quad B \subseteq \mathcal{U}.$$

- The intersection of two sets A, B is

$$A \cap B = \{ x \in \mathcal{U} \mid x \in A \text{ AND } x \in B \}$$



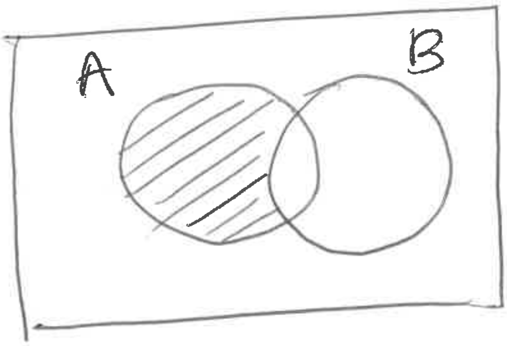
DEFINITION: A and B are DISJOINT if $A \cap B = \emptyset$.



ie: A and B have no common elements.

- The complement of a set B relative to A is the set

$$A - B = \{x \in \mathcal{U} \mid x \in A \text{ AND } x \notin B\}.$$



- In particular, the complement of a set A is

$$\overline{A} = \mathcal{U} - A = \{x \in \mathcal{U} \mid x \notin A\}$$