1

MS115

Supply and Demand

For a given good, a <u>demand function</u> relates Q_D, the quantity of the good demanded by consumers, to its price P.

We model this relationship via a linear function:

$$Q_D = aP + b$$

<u>Basic assumption</u>: As the price of the good increases, the consumer demand decreases.

This tells us that the slope of the demand function is negative,

i.e. $Q_D = aP + b$ for $\underline{a < 0}$

Moreover, as we wish to model the real-world scenario where $Q_D \geq 0$, we must have that $b > 0$

Thus $\underline{Q_D = aP + b \text{ for } a < 0 \text{ and } b > 0}$

Eg. The function $Q_D = -2P + 10$ is a typical demand function.

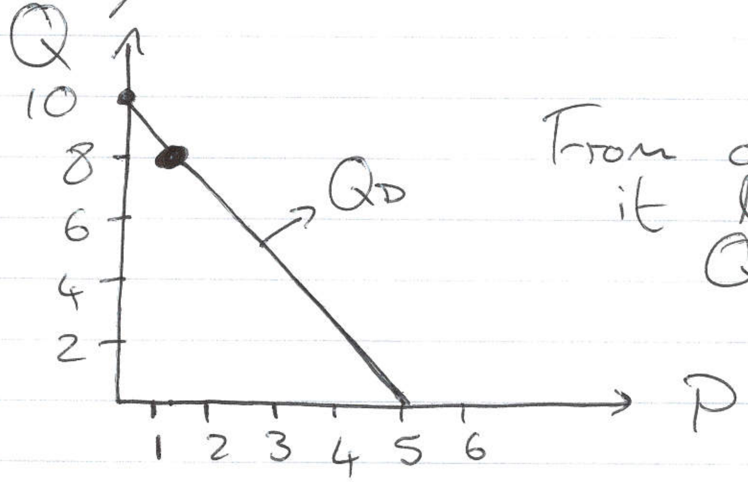
Looking at this example, we have slope -2 and vertical intercept 10.

②

As before, we can plot its straight-line graph by identifying any 2 points on the graph.

Eg. For $P = 0$, $Q_D = -2(0) + 10 = 10$, our vertical intercept.

For $P = 1$, $Q_D = -2(1) + 10 = 8$.
So our graph is the straight line through $(0, 10)$ and $(1, 8)$:



From our graph, it looks like $Q_D = 0$ when $P = 5$.

We can confirm this by solving $Q_D = 0$:

$$-2P + 10 = 0 \Rightarrow 2P = 10 \Rightarrow P = 5$$

- A <u>supply function</u> relates Q_S, the quantity of a good supplied by producers, to its price.

Again, we model via a linear function, with $Q_S = aP + b$, for $a > 0$ and $b \geq 0$.

Here, $a > 0$ to reflect the fact that an increase in price will trigger increased production,

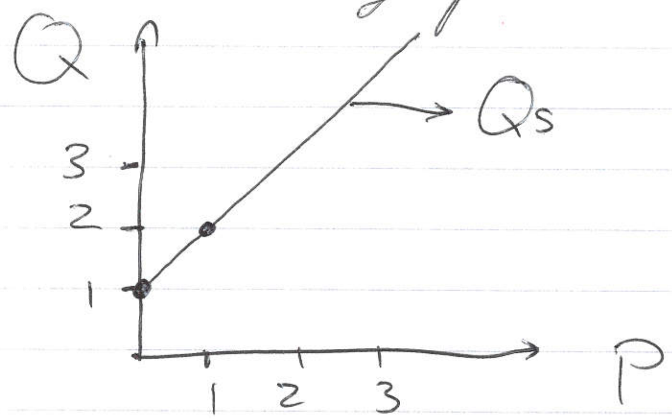
3

and $b \geq 0$ to ensure that we are dealing with $Q_s \geq 0$.

Eg. The function $Q_s = P + 1$ is a typical supply function.

For $P = 0$, we have $Q_s = 1$ and for $P = 1$, we have $Q_s = 1 + 1 = 2$.

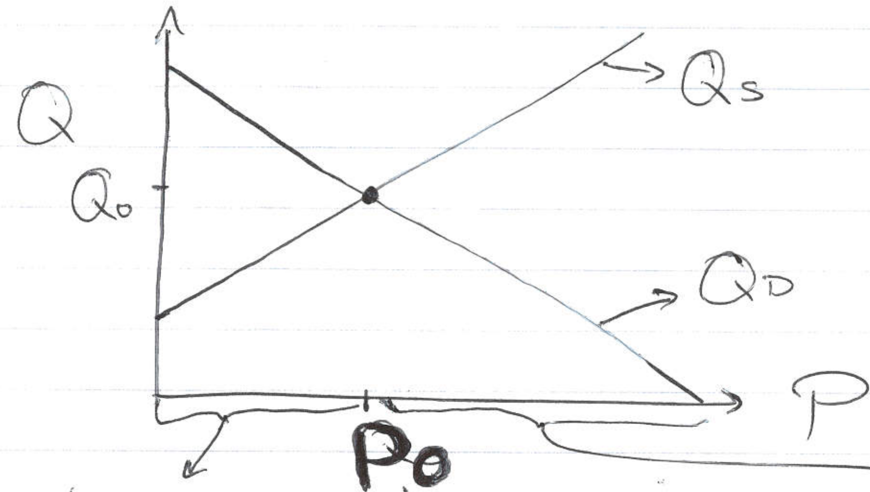
Thus, it has graph:



At <u>market equilibrium</u>, we have that $Q_D = Q_s$,

i.e. the quantity of the good that is produced exactly meets the consumer demand.

Thus, market equilibrium occurs at the point of intersection of the supply and <u>demand functions</u>, i.e. at (P_0, Q_0), the <u>equilibrium price</u> P_0 and <u>equilibrium quantity</u> Q_0.



We have $Q_D > Q_s$ for $0 \leq P < P_0$

We have $Q_D < Q_s$ for $P > P_0$

We have $Q_D = Q_s$ for $P = P_0$

- We can find the equilibrium price and quantity by solving the associated system of simultaneous equations:

Eg. Find the equilibrium price and quantity for

$$Q_D = -2P + 10 \quad \text{and} \quad Q_s = P + 1.$$

At equilibrium, we have $Q_D = Q_s$. Letting $Q_D = Q_s = Q$, we solve

$$Q = -2P + 10 \qquad (1)$$

and $$Q = P + 1 \qquad (2)$$

$(2) - (1)$: $0 = P - (-2P) + 1 - 10$

$\Rightarrow 3P = 9 \Rightarrow P = 3$

Thus $Q = (3) + 1 = 4$. Hence $P_0 = 3$ and $Q_0 = 4$

5

- We can use the demand function to model total revenue, & hence profit.

Assuming the producer can meet the consumer demand for a good (i.e. $Q_S \geq Q_D$), then the total revenue they receive is given by $\underline{TR = P \times Q_D}$,

i.e. price times number of items sold.

Subtracting a linear total cost function TC from the total revenue function gives the profit function π, which we can express as a quadratic function in Q_D,

$$\text{i.e. } \pi = aQ^2 + bQ + c$$

$$\text{for some } a, b, c \in \mathbb{R}.$$

Eg. Given $Q_D = -2P + 10$ and $TC = 2Q_D + 4$, we have that

$$TR = P \times Q_D.$$

Let's express TR as a function of Q_D: To do this, we express P as a function of Q_D and use $TR = P \times Q_D$:

$$\text{As } Q_D = -2P + 10 \Rightarrow 2P = -Q_D + 10$$

$$\Rightarrow P = -\frac{Q_D}{2} + 5$$

6

Hence $TR = P \times Q_D = \left(-\frac{Q_D}{2} + 5\right) Q_D$

$$= -\frac{Q_D^2}{2} + 5Q_D.$$

Hence, our profit function is

$$\Pi = TR - TC = -\frac{Q_D^2}{2} + 5Q_D - (2Q_D + 4)$$

i.e. $\Pi = -\frac{Q_D^2}{2} + 3Q_D - 4$