MS115 Mathematics for Enterprise Computing

Tutorial Sheet 1 Solutions

1. (*i*)

P	Q	R	$P \vee Q$	$Q \vee R$	$(P \vee Q) \vee R$	$P \vee (Q \vee R)$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	T	T
T	F	F	T	F	T	T
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	F	T	T	T
F	F	F	F	F	F	F

(*ii*)

P	Q	R	$P \wedge Q$	$Q \wedge R$	$(P \wedge Q) \wedge R$	$P \wedge (Q \wedge R)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	F	T	F	F
F	T	F	F	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

(*iii*)

P	Q	R	$Q \wedge R$	$P \vee Q$	$P \vee R$	$P \vee (Q \wedge R)$	$(P \vee Q) \wedge (P \vee R)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	T	F	F	F
F	F	T	F	F	T	F	F
F	F	F	F	F	F	F	F

(*iv*)

P	Q	R	$Q \vee R$	$P \wedge Q$	$P \wedge R$	$P \wedge (Q \vee R)$	$(P \wedge Q) \vee (P \wedge R)$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	T	F	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

Note. In class, we used the expression $P \vee Q \vee R$ without using brackets. This is valid because of the "associativity" property of the OR operator, established in (*i*).

2.

P	Q	not P	$P \Rightarrow Q$	(not P) $\vee Q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

3. (i) One example is the following:

   P : Today is Friday.

   Q : Tomorrow is not a school day.

   Clearly $P \Rightarrow Q$ is true, but $Q \Rightarrow P$ is not true in every case: today being Saturday is a case where Q holds.

   Another example is the following:

   P : x is an even number.

   Q : $2x$ is an even number.

   Clearly $P \Rightarrow Q$ is true for every even number x: if $x = 2k$ for some k then $2x = 2(2k)$, which is even.

   However, $Q \Rightarrow P$ is not true in every case: 2(3) is even but 3 is not even.

   (ii) Considering the associated truth tables, we note that if $Q \Rightarrow P$ is not logically true, then we must be in the situation where Q is true and P is false. Moreover, if P is false, we are guaranteed that $P \Rightarrow Q$ is logically true.

4. (i) Great Danes are large dogs and I have a Great Dane, or I have lots of money and I do not have a Great Dane.

   (ii) Here's an argument:

   - As $(P \wedge R) \vee (Q \wedge \text{not } R)$ is false, we know that $P \wedge R$ is false and $Q \wedge \text{not } R$ is false.
   - As $P \wedge R$ is false and P is true (we've assumed this), it follows that R is false.
   - As R is false, not R is true.
   - As $Q \wedge \text{not } R$ is false, it follows that Q is false.
     Thus R is false and Q is false.