MS115 Mathematics for Enterprise Computing

Tutorial Sheet 10 Solutions

1. 60% of the students in a class are male and 40% are female. Of the male students, 55% can program in at least one computer language, while the proportion of the female students that can program is 50%. Suppose a student is randomly selected from the class.

   (*i*) What is the probability that the selected student is male?

   Let M be the event that the selected student is male. As the choice is random, whereby every student is equally likely to be chosen, we have that $p(M) = 0.6$.

   (*ii*) What is the probability that the selected student can program, given that they are male?

   Let P be the event that the selected student can program.

   We are told that 55% of the male students can program. Hence, given that the randomly-selected student is male, there is a 55% chance that they can program. Hence, we have that $p(P|M) = 0.55$.

   (*iii*) What is the probability that the selected student cannot program, given that they are male?

   We are told that 55% of the male students can program. Hence, given that the randomly-selected student is male, there is a 45% chance that they cannot program. Hence, we have that $p\left(\overline{P}|M\right) = 0.45$.

   (*iv*) What is the probability that the selected student is female and can program?

   Let F be the event that the selected student is female. By the definition of conditional probability, we have that

   $$p(P|F) = \frac{p(P \cap F)}{p(F)}.$$

   Rearranging this expression, we have that

   $$p(P \cap F) = p(P|F)p(F) = (0.5)(0.4) = 0.2.$$

   (*v*) What is the probability that the selected student can program?

   The selected student is either male or female. Hence, the event P that the student can program can be expressed as

   $$P = (P \cap M) \cup (P \cap F),$$

i.e. the student can program and is male or the student can program and is female.

Hence, by our Addition Rule for probabilities, we have that

$$p(P) = p(P \cap M) + p(P \cap F).$$

As above, by rearranging the definition of conditional probability, we have that

$$p(P \cap M) = p(P|M)p(M) = (0.55)(0.6) = 0.33$$

and

$$p(P \cap F) = p(P|F)p(F)(0.5)(0.4) = 0.2.$$

Hence, $p(P) = 0.33 + 0.2 = 0.53$.

(vi) What is the probability that the person selected student is male, given that they can program?

By the definition of conditional probability, we have that

$$p(M|P) = \frac{p(M \cap P)}{p(P)}.$$

As above, we know that $p(P \cap M) = p(P|M)p(M) = (0.55)(0.6) = 0.33$ and $p(P) = 0.53$. Hence, we have that

$$p(M|P) = \frac{p(M \cap P)}{p(P)} = \frac{0.33}{0.53} = 0.62 \text{ (to 2 decimal places).}$$

(vii) What is the probability that the person selected student is female, given that they can program?

We can calculate this by arguing as in the previous part.

Alternatively, since M and F are complementary events, we can use of Complement Rule for probabilities to establish that

$$p(F|P) = p\left(\overline{M}|P\right) = 1 - p(M|P) = 1 - 0.62 = 0.38 \text{ (to 2 decimal places).}$$

2. In a widget factory 30%, 50% and 20% of production is done on machines 1, 2 and 3 respectively. It is known that 4%, 2% and 3% of the respective output of these machines is defective. What is the probability that a randomly selected widget is defective?

Let D be the event the randomly-selected widget is defective.

For $i = 1, 2, 3$, let M_i be the event that the randomly-selected widget is manufactured by machine i. Arguing as in 1. (v) above, we have that

$$\begin{aligned} P(D) &= P(D \cap M_1) + P(D \cap M_2) + P(D \cap M_3) \\ &= P(D|M_1)P(M_1) + P(D|M_2)P(M_2) + P(D|M_3)P(M_3) \\ &= 0.04 \times 0.3 + 0.02 \times 0.5 + 0.03 \times 0.2 = 0.028 \end{aligned}$$