MS115 Mathematics for Enterprise Computing

Tutorial Sheet 3

1. List the elements of the following sets:

   (i) $\{n \in \mathbb{N} \mid n > 3 \text{ and } n^2 < 100\} = \{4, 5, 6, 7, 8, 9\}$

   (ii) $\{x \in \mathbb{Z} \mid x^2 = 4 \text{ or } 0 < x < 4\} = \{-2, 2, 1, 3\}$

   (iii) $\{(n-1)^2 + 2 \mid n \in \{2, 3, 4\}\} = \{3, 6, 11\}$

2. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ be the universal set and consider the following subsets of U:

   $$A = \{1, 3, 5\}, \quad B = \{1, 3, 5, 7, 9\}, \quad C = \{2, 3, 5, 7\}, \quad D = \emptyset$$

   Determine the following sets:

   (i) $A \cup B = \{1, 3, 5, 7, 9\}$, (ii) $C \cup D = \{2, 3, 5, 7\}$, (iii) $C \cap D = \emptyset$,

   (iv) $B - A = \{7, 9\}$, (v) $A - B = \emptyset$, (vi) $(A \cup C) - B = \{2\}$

   (vii) $\overline{A} = \{2, 4, 6, 7, 8, 9\}$ $(viii)$ $B - (\overline{A \cap C}) = \{3, 5\}$.

3. Determine whether the sets A, B and C are pairwise disjoint in each of the following cases:

   (i) $A = \{3, 5\}$, $B = \{1, 4, 6\}$, $C = \{2\}$.
   Yes: $A \cap B = \emptyset$, $A \cap C = \emptyset$, $B \cap C = \emptyset$.

   (ii) $A = \{2, 4, 6\}$, $B = \{3, 7\}$, $C = \{4, 5\}$.
   No: $A \cap B = \emptyset$, $B \cap C = \emptyset$, but $A \cap C = \{4\}$.

4. Let $A = \{T, F\}$ and consider the set $A^3 = A \times A \times A$.

   (i) $|A^3| = |A|^3 = 2^3 = 8$.

   (ii) $(T, T, T), (T, T, F), (T, F, T), (T, F, F), (F, T, T), (F, T, F), (F, F, T), (F, F, F)$.

5. (i) Using the Inclusion-Exclusion principle, we have that

   $$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|.$$

   Thus

   $$|A \cup B \cup C| = 16 + 23 + 30 - 5 - 2 - 15 + 0 = 47.$$

(*ii*) Labelling the sets in a Venn diagram with A, B and C (for accounting, business and computing respectively), we recognise that the students that study computing only are in C and not in A and not in B, and thus are in the region $C \cap \overline{A} \cap \overline{B}$.

As

$$|C| = |C \cap \overline{A} \cap \overline{B}| + |C \cap A \cap \overline{B}| + |C \cap B \cap \overline{A}| + |C \cap A \cap B|,$$

we have that

$$|C \cap \overline{A} \cap \overline{B}| = |C| - |C \cap A \cap \overline{B}| - |C \cap B \cap \overline{A}| - |C \cap A \cap B|,$$

and thus

$$|C \cap \overline{A} \cap \overline{B}| = 30 - 2 - 15 - 0 = 13.$$