MS115 Mathematics for Enterprise Computing

Tutorial Sheet 5 Solutions

1. (i) We determine the inverse by expressing the input to the function in terms of its output. Letting y denote the output of the function, we thus express x in terms of y:

$$y = 2x + 2 \Rightarrow -2x + y = 2 \Rightarrow -2x = -y + 2 \Rightarrow x = \frac{-1}{2}(-y + 2)$$

Thus $x = \frac{y}{2} - 1$. Hence $g(y) = \frac{y}{2} - 1$ is the inverse of $f(x)$.

(ii) Similarly, we express x in terms of y, where y is the output of the function:

$$y = -3x + 4 \Rightarrow 3x + y = 4 \Rightarrow 3x = -y + 4 \Rightarrow x = \frac{1}{3}(-y + 4)$$

Thus $x = \frac{-y}{3} + \frac{4}{3}$. Hence $g(y) = \frac{-y}{3} + \frac{4}{3}$ is the inverse of $f(x)$.

(iii) Similarly, we express x in terms of y, where y is the output of the function:

$$y = \frac{2x + 2}{3x - 1} \Rightarrow 3xy - y = 2x + 2 \Rightarrow 3xy - 2x = y + 2 \Rightarrow x(3y - 2) = y + 2.$$

Thus $x = \frac{y+2}{3y-2}$. Hence $g(y) = \frac{y+2}{3y-2}$ is the inverse of $f(x)$.

(iv) Similarly, we express x in terms of y, where y is the output of the function:

$$y = \frac{x + 4}{-2x + 1} \Rightarrow -2xy + y = x + 4 \Rightarrow -2xy - x = -y + 4 \Rightarrow x(-2y - 1) = -y + 4.$$

Thus $x = \frac{-y+4}{-2y-1}$. Hence $g(y) = \frac{-y+4}{-2y-1}$ is the inverse of $f(x)$.

2. (i) $f(x) = \frac{x-1}{x+5}$ is defined for all $x \neq -5$. Thus, the largest domain on which f is defined is $\mathbb{R} - \{-5\}$.

(ii) The equation $\frac{x-1}{x+5} = a$ does not have a solution for $a = 1$, as

$$\frac{x - 1}{x + 5} = 1 \Rightarrow x - 1 = x + 5 \Rightarrow -1 = 5, \text{ a contradiction.}$$

The range of f is $\mathbb{R} - \{1\}$, as for all other values of a the equation $\frac{x-1}{x+5} = a$ has a solution:

$$\frac{x - 1}{x + 5} = a \Rightarrow x - 1 = ax + 5a \Rightarrow x - ax = 1 + 5a \Rightarrow x(1 - a) = 1 + 5a,$$

and this equation has solution

$$x = \frac{1 + 5a}{1 - a}.$$

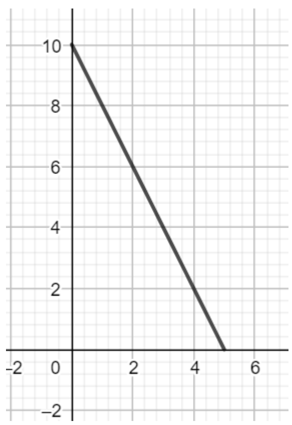
(*iii*) Expressing x in terms of y, where y is the output of the function:

$$y = \frac{x-1}{x+5} \Rightarrow xy+5y = x-1 \Rightarrow xy-x = -5y-1 \Rightarrow x(y-1) = -5y-1.$$

Thus $x = \frac{-5y-1}{y-1}$. Hence $g(y) = \frac{-5y-1}{y-1}$ is the inverse of $f(x)$.

3. (*i*) $2x + y = 10 \Rightarrow y = -2x + 10$, whereby the line has slope -2 and y-intercept 10.

(*ii*) $(0, 10)$ and $(1, 8)$ are two such points. We can determine a point on the line by choosing an x-value and determining the corresponding y-value in accordance with the equation $y = -2x + 10$.

(*iii*) 

(*iv*) Solving $y = 0$ gives $-2x + 10 = 0 \Rightarrow -2x = -10 \Rightarrow x = 5$.

4. Determine the point of intersection of the following pairs of straight lines:

(*i*) For $y = x + 2$ and $y = 3x$, we have that $x + 2 = 3x$, whereby $x = 1$ and hence $y = 3$.

(*ii*) For $2y = x + 2$ and $y = -2x + 7$, we have that $2y = x + 2$ and $2y = -4x + 14$, whereby $x + 2 = -4x + 14$. Thus, $5x = 12$ and hence $x = \frac{12}{5}$. Thus, $y = -2\left(\frac{12}{5}\right) + 7 = \frac{11}{5}$.