MS115 Mathematics for Enterprise Computing

Tutorial Sheet 6

1. Given the demand function $Q_D = -4P + 10$, determine the minimum price at which demand equals zero.

   To determine the horizontal intercept, we solve $Q_D = 0$. Solving $-4P+10 = 0$ gives $4P = 10$ and thus $P = 2.5$. Thus, $P = 2.5$ is the minimum price at which the demand is zero.

   (We consider the demand Q_D to be zero for all prices $P > 2.5$, as we do not consider the possibility of negative demand).

2. Consider the following demand and supply functions of a particular good:

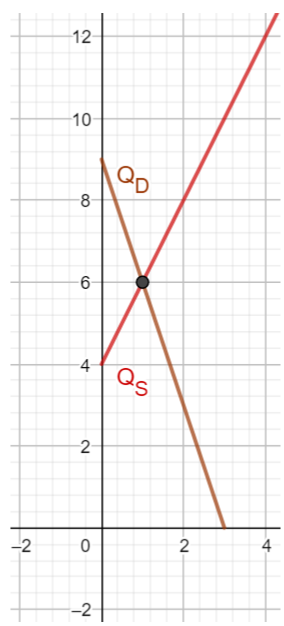
$$Q_D = -3P + 9 \qquad \text{and} \qquad Q_S = 2P + 4.$$

   (i) Identify the slopes and vertical intercepts of these functions.

   The demand function has slope -3 and vertical intercept 9
   (the vertical intercept is the value of Q_D when $P = 0$).
   The supply function has slope 2 and vertical intercept 4
   (the vertical intercept is the value of Q_D when $P = 0$).

   (ii) Sketch the graphs of these functions on the same pair of axes.

   By finding two points on each straight-line graph, we can sketch the following graphs on the same pair of axes:



(*iii*) Determine the equilibrium price and quantity of this good.

At market equilibrium we have that $Q_D = Q_S$. For $Q_D = Q_S = Q$, we thus solve the following system of linear equations:

$$\begin{aligned} Q &= -3P + 9 \qquad (1) \\ Q &= 2P + 4. \qquad (2) \end{aligned}$$

Subtracting equation (1) from equation (2) gives that $5P = 5$ whereby $P = 1$ and thus $Q = 2(1) + 4 = 6$. Hence, the equilibrium price is $P_0 = 1$ and the equilibrium quantity is $Q_0 = 6$.

3. Determine the equilibrium price and quantity of a good whose demand are supply functions are as follows:

$$Q_D = -P + 8 \qquad \text{and} \qquad Q_S = 2P + 2.$$

At market equilibrium we have that $Q_D = Q_S$. For $Q_D = Q_S = Q$, we thus solve the following system of linear equations:

$$\begin{aligned} Q &= -P + 8 \qquad (3) \\ Q &= 2P + 2. \qquad (4) \end{aligned}$$

Subtracting equation (1) from equation (2) gives that $3P = 6$ whereby $P = 2$ and thus $Q = 2(1) + 2 = 6$. Hence, the equilibrium price is $P_0 = 2$ and the equilibrium quantity is $Q_0 = 6$.

4. Consider the following demand and supply functions of a particular good:

$$Q_D = -5P + 15 \qquad \text{and} \qquad Q_S = 3P + 5.$$

(*i*) Determine the range of prices for which demand exceeds supply.

Considering the graphs of the supply and demand functions, we note that $Q_D > Q_S$ when $0 \leq P < P_0$, i.e. when P is less than the equilibrium price.
Solving for the equilibrium price as in the above examples, we obtain that $P_0 = 1.25$. Hence, demand exceeds supply when $0 \leq P < 1.25$.

(*ii*) Determine the range of prices for which supply exceeds demand.

Considering the graphs of the supply and demand functions, we note that $Q_D < Q_S$ when $P > P_0$, i.e. when P is greater than the equilibrium price.
Given that the equilibrium price is $P_0 = 1.25$, as discussed above, supply exceeds demand when $P > 1.25$.

5. Consider the following demand function of a particular good:

$$Q_D = -2P + 13.$$

(i) Invert the demand function to express P as a function of Q_D.

Expressing the input P in terms of the output Q_D gives

$$-2P + 13 = Q_D \quad \Rightarrow \quad -2P = Q_D - 13 \quad \Rightarrow \quad P = -\frac{Q_D}{2} + \frac{13}{2}.$$

Hence

$$P = -\frac{Q_D}{2} + \frac{13}{2}.$$

(ii) Express total revenue TR as a function of Q_D.

By definition, the total revenue function TR is given by $TR = P \times Q_D$. As $P = -\frac{Q_D}{2} + \frac{13}{2}$, we thus obtain that

$$TR = \left(-\frac{Q_D}{2} + \frac{13}{2}\right) Q_D = -\frac{{Q_D}^2}{2} + \left(\frac{13}{2}\right) Q_D.$$

Hence

$$TR = -\frac{{Q_D}^2}{2} + \left(\frac{13}{2}\right) Q_D.$$

(iii) Given a total cost function $TC = \frac{Q_D}{2} + 10$, express the profit function as a quadratic function in Q_D.

The profit function is given by

$$TR - TC = -\frac{{Q_D}^2}{2} + \left(\frac{13}{2}\right) Q_D - \left(\frac{Q_D}{2} + 10\right) = -\frac{{Q_D}^2}{2} + 6Q_D - 10.$$

(iv) Determine the values of Q_D for which profit equals zero.

We must solve for Q_D such that $-\frac{{Q_D}^2}{2} + 6Q_D - 10 = 0$.
We can do this by invoking the "$-b$ formula":
Note: if $ax^2 + bx + c = 0$ has real solutions, these solutions occur at

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Here, $a = -\frac{1}{2}$, $b = 6$ and $c = -10$, whereby the solutions occur at

$$x = \frac{-6 \pm \sqrt{36 - 4\left(-\frac{1}{2}\right)(-10)}}{2\left(-\frac{1}{2}\right)} = \frac{-6 \pm \sqrt{16}}{-1} = \frac{-6 \pm 4}{-1}.$$

Thus, we have solutions at $Q_D = 6 - 4 = 2$ and $Q_D = 6 + 4 = 10$.
Hence, we have zero profit at $Q_D = 2$ and $Q_D = 10$.