MS115 Mathematics for Enterprise Computing

Tutorial Sheet 7 Solutions

1. Consider the following demand function of a particular good:

$$Q_D = -4P + 9.$$

(*i*) Invert the demand function to express P as a function of Q_D.

Expressing the input P in terms of the output Q_D gives

$$-4P + 9 = Q_D \quad \Rightarrow \quad -4P = Q_D - 9 \quad \Rightarrow \quad P = -\frac{Q_D}{4} + \frac{9}{4}.$$

Hence

$$P = -\frac{Q_D}{4} + \frac{9}{4}.$$

(*ii*) Express total revenue TR as a function of Q_D.

By definition, the total revenue function TR is given by $TR = P \times Q_D$. As $P = -\frac{Q_D}{4} + \frac{9}{4}$, we thus obtain that

$$TR = \left(-\frac{Q_D}{4} + \frac{9}{4}\right) Q_D = -\frac{{Q_D}^2}{4} + \left(\frac{9}{4}\right) Q_D.$$

(*iii*) Given a total cost function $TC = \left(\frac{1}{4}\right) Q_D + 3$, express the profit function as a quadratic function in Q_D.

The profit function is given by

$$TR - TC = -\frac{{Q_D}^2}{4} + \left(\frac{9}{4}\right) Q_D - \left(\frac{Q_D}{4} + 3\right) = -\frac{{Q_D}^2}{4} + 2Q_D - 3.$$

(*iv*) Determine the values of Q_D for which profit equals zero.

We must solve for Q_D such that $-\frac{{Q_D}^2}{4} + 2Q_D - 3 = 0$.
We can do this by invoking the "$-b$ formula":
Note: if $ax^2 + bx + c = 0$ has real solutions, these solutions occur at

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Here, $a = -\frac{1}{4}$, $b = 2$ and $c = -3$, whereby the solutions occur at

$$x = \frac{-2 \pm \sqrt{4 - 4\left(-\frac{1}{4}\right)(-3)}}{2\left(-\frac{1}{4}\right)} = \frac{-2 \pm \sqrt{1}}{-\frac{1}{2}} = \frac{-2 \pm 1}{-\frac{1}{2}}.$$

Thus, we have solutions at $Q_D = 2$ and $Q_D = 6$.
Hence, we have zero profit at $Q_D = 2$ and $Q_D = 6$.

(v) Determine the values of Q_D for which there is a positive profit.

By considering the graph of the function, we note that we have positive profit for $2 < Q_D < 6$.

(vi) Determine the value of Q_D which results in the maximum profit.

The maximum profit occurs at

$$Q_D = -\frac{b}{2a} = -\frac{2}{2\left(-\frac{1}{4}\right)},$$

which is the midpoint between the horizontal intercepts. Thus, we have a maximum profit at $Q_D = 4$.

(vii) Determine the maximum profit.

The maximum profit is the value of Π at $Q_D = 4$. This is given by

$$\Pi(4) = -\frac{4^2}{4} + 2(4) - 3 = 1.$$

2. A man has five suits, seven shirts and three ties. How many different outfits can he put together?

An outfit is created by making a choice of suit, followed by a choice of shirt and then a choice of tie. Invoking the Product Principle of Counting, we thus have $(5)(7)(3) = 105$ possible outfits.

3. A bank P.I.N. is a number consisting of four digits.

(i) Determine the total number of bank P.I.N.s.

A P.I.N. is created by making a choice of the first, second, third and fourth digit. We have 10 possibilities for each digit (0,1,2,. . .,9), whereby there are $(10)(10)(10)(10) = 10^4 = 10,000$ possible P.I.N.s.

(ii) Determine the total number of bank P.I.N.s in which no digit is repeated.

As above, a P.I.N. is created by making a choice of the first, second, third and fourth digit. As repeated selection is not allowed, we have 10 possibilities for the first digit, 9 for the second, 8 for the third and 7 for the fourth, whereby there are $(10)(9)(8)(7) = 10^4 = 5040$ possible P.I.N.s with no repeated digits.

(*iii*) Determine the total number of bank P.I.N.s in which the first digit is non-zero and no digit is repeated.

We argue as above. As the first digit must be non-zero, we have 9 possibilities for the first digit. We also have 9 possibilities for the second digit (as it may equal zero). We have 8 possibilities for the third digit (as it must differ from the first and second) and 7 possibilities for the fourth digit, whereby there are $(9)(9)(8)(7) = 10^4 = 4536$ possible P.I.N.s that begin with a non-zero digit and contain no repeated digits.

4. There are 12 participants in a race. Determine the total number of possible podium outcomes.

We seek to determine the number of possible first-place, second-place and third-place outcomes. We can view this as choosing 3 participants from 12, where order of selection is important and repeated selection is not allowed. Thus, we have $(12)(11)(10) = 1320$ possible podium outcomes.

5. A palindrome is a sequence of letters that reads the same backwards and forwards (eg. XYYX or NAVAN or ABLEWASIEREISAWELBA).

(*i*) Determine the total number of four-letter palindromes.

We have 26 possible choices for the first letter in our four-letter sequence, and 26 possible choices for the second letter in our four-letter sequence. As the third letter must agree with the second letter, there is only one choice for the third letter in our four-letter sequence. Similarly, the last letter must agree with the first letter. Hence we have $(26)(26) = 676$ possible four-letter palindromes.

(*ii*) Determine the total number of five-letter palindromes.

We have 26 possible choices for the first letter in our five-letter sequence, 26 possible choices for the second letter in our five-letter sequence and 26 possible choices for the third letter in our five-letter sequence. As the fourth letter must agree with the second letter, there is only one choice for the fourth letter in our five-letter sequence. Similarly, the last letter must agree with the first letter. Hence we have $(26)(26)(26) = 17,576$ possible five-letter palindromes.