MS115 Mathematics for Enterprise Computing

Tutorial Sheet 8 Solutions

1. Consider the five letters in the word "bread".

   (i) Determine the number of ways of arranging these five letters in order, i.e. determine the number of five-letter strings that can be formed using distinct letters.

   We know that n distinct objects can be ordered in $n!$ ways. Hence, we can order the letters b,r,e,a,d in $5! = (5)(4)(3)(2)(1) = 120$ ways.

   (ii) Determine the number of four-letter strings that can be formed using distinct letters.

   In forming a string consisting of four distinct letters, we are selecting $k = 4$ objects in order from $n = 5$ distinct objects without repeated selection. Hence, we have $\frac{n!}{(n-k)!} = \frac{5!}{(5-4)!} = \frac{5!}{1!} = 120$ ways.

2. Consider the set $S = \{a, b, c, d, e\}$.

   (i) Determine the total number of subsets of S.

   By the Product Principle, we know that there are $2^{|S|} = 2^5 = 32$ possible subsets of S.

   (ii) Determine the total number of subsets T of S such that $|T| = 3$.

   In forming a subset of size k from a set of size n, we are choosing k elements without repeated selection and without regard to order from n. Hence, we have $\binom{n}{k}$ subsets of size k. Thus, here we have

   $$\binom{5}{3} = \frac{5!}{2!3!} = \frac{(5)(4)}{(2)(1)} = 10$$

   subsets of size 3.

3. A committee of 5 people is formed from a panel of 15 candidates, 9 of whom are male and 6 of whom are female.

   (i) Determine the total number of possible committees that may be formed.

   We are choosing 5 people from 15, without regard to the order of choice and without repeated selection. Hence, there are

   $$\binom{15}{5} = \frac{15!}{10!5!} = \frac{(15)(14)(13)(12)(11)}{(5)(4)(3)(2)(1)} = 3,003$$

   possible committees.

(*ii*) Determine the total number of possible committees that contain no males.

Such a committee consists of 5 females. Hence, we are choosing 5 females from 6, without regard to the order of choice and without repeated selection. Hence, there are

$$\binom{6}{5} = \frac{6!}{5!1!} = \frac{6}{1} = 6$$

possible committees that contain no males.

(*iii*) Determine the total number of possible committees that contain two females and three males.

Such a committee consists of 2 females and 3 males. Hence, we are choosing 2 females from 6 and 3 males from 9, without regard to the order of choice and without repeated selection in both cases. Invoking the Product Principle, we have

$$\binom{6}{2}\binom{9}{3} = (15)(84) = 1,260$$

possible committees that contain two females and three males.

(*iv*) Determine the total number of possible committees that contain at least one male and at least one female.

Here, it is easier to count the number of committees that do not contain at least one male and at least one female, and then to subtract this number from the total number of possible committees to determine our answer.

Arguing as in part (*ii*), there are $\binom{6}{5} = 6$ committees that contain no males and $\binom{9}{5} = 126$ committees that contain no females. Invoking the Addition Principle, there are $6 + 126 = 132$ committees that do not contain at least one male and at least one female. Hence, using our answer from part (*i*), there are $3,003 - 132 = 2,871$ committees that contain at least one male and at least one female.

4. The main draw in the Irish national lottery consists of selecting 6 numbered balls from a total of 47.

(*i*) Determine the total number of possible outcomes of this draw.

A draw is a selection of 6 numbered balls from 47, without regard to the order of choice and without repeated selection. Hence, we have

$$\binom{47}{6} = \frac{47!}{41!6!} = \frac{(47)(46)(45)(44)(43)(42)}{(6)(5)(4)(3)(2)(1)} = 10,737,573$$

possible outcomes.

(*ii*) Suppose that your lottery ticket consists of one selection of 6 numbers from the 47 possibilities. Consider the event that exactly 3 of the numbers on your ticket match those on the drawn balls. In how many ways can this event occur?

This event occurs if 3 of our 6 numbers match 3 of the 6 numbers in the drawn outcome and 3 of our numbers do not match any of the 3 remaining numbers in the drawn outcome. Thus, we can viewing this event as occurring if we select 3 (correct) numbers from the 6 numbers in the drawn outcome and select 3 (incorrect) numbers from the 41 that do not appear in the drawn outcome. Invoking the Product Principle, we thus have

$$\binom{6}{3}\binom{41}{3} = (20)(10,660) = 213,200$$

such outcomes.

5. A governmental cabinet consists of 22 members. A secret ballot is taken, whereby each member must choose one of two options: "Deal" or "No deal". The 22 votes are pooled together and counted. Determine the number of possible outcomes to this vote.

We may view this problem as selecting $k = 22$ objects from $n = 2$ distinct objects ("Deal" and "No deal") with repeated selection allowed but without regard to the order of selection. Thus, using our "objects and separators" formula, we have

$$\binom{k+n-1}{n-1} = \binom{23}{1} = \frac{23!}{22!1!} = 23$$

possible outcomes.