MS115 Mathematics for Enterprise Computing

Tutorial Sheet 9 Solutions

1. Consider the ten letters in the word $ENTERPRISE$.

   (i) Per class, add subscripts to these letters where necessary to produce a string of ten distinct letters.

   Using subscripts to distinguish between repeated letters, we have $E_1NTE_2R_1PR_2ISE_3$.

   (ii) Per class, strings that coincide upon the removal of subscripts are equivalent. List all the elements in an equivalence class of your choice.

   Let's consider, for example, the equivalence class of $E_1E_2E_3TNIPSR_1R_2$.
   The following strings are equivalent to $E_1E_2E_3TNIPSR_1R_2$:

   - $E_1E_2E_3TNIPSR_1R_2$ itself (due to reflexivity),
   - $E_1E_3E_2TNIPSR_1R_2$
   - $E_2E_1E_3TNIPSR_1R_2$
   - $E_2E_3E_1TNIPSR_1R_2$
   - $E_3E_1E_2TNIPSR_1R_2$
   - $E_3E_2E_1TNIPSR_1R_2$
   - $E_1E_2E_3TNIPSR_2R_1$
   - $E_1E_3E_2TNIPSR_2R_1$
   - $E_2E_1E_3TNIPSR_2R_1$
   - $E_2E_3E_1TNIPSR_2R_1$
   - $E_3E_1E_2TNIPSR_2R_1$
   - $E_3E_2E_1TNIPSR_2R_1$

   (iii) Determine the number of elements in each equivalence class.

   As above, we have $3! \times 2! = 12$ elements in each equivalence class, according the our ordering of the 3 E subscripts and the 2 R subscripts.

   (iv) Determine the number of ways of ordering the letters in the word $ENTERPRISE$.

   As each equivalence class represents one ordering of the letters in $ENTERPRISE$, we seek to determine the number of equivalence classes. As above, each equivalence class contains 12 elements. Since the equivalence classes form a partition of the set of orderings of the ten distinct subscripted letters, we have that $12x = 10!$, where x represents the number of equivalence classes. Hence, our answer is given by

$$x = \frac{10!}{12} = 302,400.$$

2. Determine the number of ways of ordering the fifteen letters in the string "SUPPLYANDDEMAND".

   Arguing as in question 1, we have

$$\frac{15!}{2! \times 2! \times 2! \times 3!} = 27,243,216,000 \text{ orderings.}$$

3. Twelve students are to be divided into three groups A, B, C of equal size. Determine the total number of possible outcomes of this process.

   We can interpret this process as ordering the letters A, B and C with repetition. We can see this by lining up the 12 students and assigning each student one of the letters in a way so that we have 4 A's, 4 B's and 4 C's. Thus, we seek to determine the number of ways of ordering the letters in the string $AAAABBBBCCCC$. As above, we can do this in

$$\frac{12!}{4! \times 4! \times 4!} = 34,650 \text{ ways.}$$

4. An experiment consists of randomly choosing a lowercase letter from the English alphabet.

   (*i*) List the outcomes that make up the sample space Ω.

   $\Omega = \{a, b, c, d, \ldots, x, y, z\}$.

   (*ii*) List the outcomes that make up the event E that the randomly-chosen letter is a vowel.

   $E = \{a, e, i, o, u\}$.

   (*iii*) Determine $p(E)$, the probability of E.

   As we have a random selection, every letter has an equal probability of being selected. Hence, each letter is selected with probability $\frac{1}{26}$. Now

$$p(E) = p(a)+p(e)+p(i)+p(o)+p(u) = \frac{1}{26}+\frac{1}{26}+\frac{1}{26}+\frac{1}{26}+\frac{1}{26} = \frac{5}{26}.$$

   (*iv*) Determine $p\left(\overline{E}\right)$, the probability of $\overline{E}$.

   $\overline{E} = \{b, c, d, f, \ldots, x, y, z\}$, a set containing 21 outcomes. As above, since each letter is selected with probability $\frac{1}{26}$, we have

$$p(\overline{E}) = p(b) + p(c) + \ldots + p(y) + p(z) = \frac{21}{26}.$$

5. A pair of dice is thrown and the sum of their values is recorded.

   (i) List the outcomes in the sample space Ω.
   $\Omega = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

   (ii) List the outcomes that make up the following events:

   A is the event that the sum is greater than 7,
   B is the event that the sum is an odd number.

   $A = \{8, 9, 10, 11, 12\}$ and $B = \{3, 5, 7, 9, 11\}$.

   (iii) List the elements of the following events: $\overline{A}$, $A \cap B$, $A \cup B$.
   $\overline{A} = \{2, 3, 4, 5, 6, 7\}$, $A \cap B = \{9, 11\}$ and $A \cup B = \{3, 5, 7, 8, 9, 10, 11, 12\}$.

   (iv) Give the probabilities of all the outcomes in the sample space Ω.

   By the Product Principle, there are 36 outcomes to the rolling of two dice, each with probability $\frac{1}{36}$.

   The sum outcome 2 arises in one way: a 1 is rolled and a 1 is rolled. By contrast, the sum outcome 3 arises in 2 ways: a 1 is rolled and a 2 is rolled or a 2 is rolled and a 1 is rolled. By counting the number of dice roll outcomes that give rise to the various sum outcomes, and summing their probabilities, we obtain the following list of probabilities for the sum outcomes:

   $p(2) = 1/36$, $p(3) = 2/36 = 1/18$, $p(4) = 3/36 = 1/12$,
   $p(5) = 4/36 = 1/9$, $p(6) = 5/36$, $p(7) = 6/36 = 1/6$,
   $p(8) = 5/36$, $p(9) = 4/36 = 1/9$, $p(10) = 3/36 = 1/12$,
   $p(11) = 2/36 = 1/18$, $p(12) = 1/36$.

   (v) Determine $p(A)$, $p(B)$ and $p(A \cap B)$.

   Summing the probabilities of the outcomes in each event, we have

   $p(A) = p(8) + p(9) + p(10) + p(11) + p(12) = \frac{5+4+3+2+1}{36} = \frac{5}{12}$,

   $p(B) = p(3) + p(5) + p(7) + p(9) + p(11) = \frac{2+4+6+4+2}{36} = \frac{1}{2}$,

   $p(A \cap B) = p(9) + p(11) = \frac{4+2}{36} = \frac{1}{6}$.

   Note that $p(A \cap B) \neq p(A) \cdot p(B)$.