MS115 Mathematics for Enterprise Computing

Tutorial Sheet 1

1. Use truth tables to show the following:

   (i) $(P \vee Q) \vee R$ is logically equivalent to $P \vee (Q \vee R)$.

   (ii) $(P \wedge Q) \wedge R$ is logically equivalent to $P \wedge (Q \wedge R)$.

   (iii) $P \vee (Q \wedge R)$ is logically equivalent to $(P \vee Q) \wedge (P \vee R)$.

   (iv) $P \wedge (Q \vee R)$ is logically equivalent to $(P \wedge Q) \vee (P \wedge R)$.

   Note. In class, we used the expression $P \vee Q \vee R$ without using brackets. This is valid because of the "associativity" property of the OR operator, established in (i).

2. We have seen that the conditional statement $P \Rightarrow Q$ is logically equivalent to its *contrapositive* not $Q \Rightarrow$ not P. Use a truth table to show that $P \Rightarrow Q$ is also logically equivalent to (not P) $\vee Q$.

3. A proposition that is true in every possible case is said to be *logically true*.

   (i) Give an example of two propositions P and Q such that $P \Rightarrow Q$ is logically true but $Q \Rightarrow P$ is not. Thus, $P \Rightarrow Q$ should be true in every case but $Q \Rightarrow P$ should be false in at least one case.

   (ii) Let P and Q be *any* propositions such that $P \Rightarrow Q$ is logically true but $Q \Rightarrow P$ is not. What are the truth values of P and Q?

4. Consider the following propositions:

   P : Great Danes are large dogs.

   Q : I have lots of money.

   R : I have a Great Dane.

   (i) Express the following compound proposition as an English sentence:

   $$(P \wedge R) \vee (Q \wedge \text{not } R)$$

   (ii) Let's agree that P is true. Suppose that $(P \wedge R) \vee (Q \wedge \text{not } R)$ is false. Use a logical argument or a truth table to determine the truth values of Q and R.