MS115 Mathematics for Enterprise Computing

Tutorial Sheet 2

1. Per class, De Morgan's Laws state the following:

   (i) not $(P \wedge Q) \equiv$ not $P \vee$ not Q, $\quad (ii)$ not $(P \vee Q) \equiv$ not $P \wedge$ not Q.

   Use a truth table to prove (ii).

2. (i) Suppose that n and m are integers. Give a direct proof that the following proposition is logically true:

   If n and m are even integers, then $n + m$ is an even integer.

   (ii) Suppose that n is an integer. Give a contrapositive proof that the following proposition is logically true:

   If n^2 is an even integer, then n is an even integer.

3. Use induction to prove that the following predicate is true for all $n \geqslant 1$:

   $$P(n): \quad 1 + 3 + 5 + \ldots + (2n - 1) = n^2.$$

4. Use induction to prove the following statements hold for all $n \geq 1$:

   (i) If a is a positive real number with $a \neq 1$, then

   $$1 + a + a^2 + \ldots + a^{n-1} = \frac{1 - a^n}{1 - a}$$

   (ii) $(n^3 - n)$ is divisible by 3
   Hint: Recall that, for m and n integers, m is divisible by n means that there exists an integer k such that $m = nk$.