DUBLIN CITY UNIVERSITY

MS 221 - Make-up Class Test (1)

Programme	Name	I.D. Number

DATE: **25 October 2018**

Programme:
B. Sc. in Applied Physics
B. Sc. in Physics with Astronomy
B. Sc. in Physics with Biomedical Sciences
Bachelor of Arts (BAJH)
B. Sc. in Physical Education with Mathematics (Year 3)
B. Sc. in Science Education (Year 3)

YEAR: 2 and 3

SUBJECT: **Calculus Of Several Variables (MS 221)**

TIME ALLOWED: 50 Minutes

EXAMINER: Dr. M. Clancy

INSTRUCTIONS: **Answer ALL Questions**

DO NOT TURN OVER THIS PAGE UNTIL YOU ARE TOLD THAT YOU MAY DO SO

This examination paper contains 6 pages and 8 questions. This cover page is page 1.

QUESTION 1

Is it possible to assign a value to $f(-3/2)$ in such a way that the function

$$f:(-\infty,0)\to\mathbb{R}:x\mapsto\begin{cases}\dfrac{2x^2-x-6}{2x^2+x-3} & \text{when} \quad x\neq-\dfrac{3}{2}\\[2ex] f(-3/2) & \text{when} \quad x=-\dfrac{3}{2}\end{cases}$$

is continuous at $x=-3/2$? Justify your answer.

[5 marks]

QUESTION 2

Calculate the derivative: $\dfrac{d}{dx}\displaystyle\int_0^{x^2+x}\frac{\sin t^5}{1+t^6}\,dt\,.$

[5 marks]

QUESTION 3

Calculate the **angle** θ (measured in degrees) between the vectors $\boldsymbol{u}$ and $\boldsymbol{v} \in \mathbb{R}^3$ where:

$$\boldsymbol{u} = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} \quad \text{and} \quad \boldsymbol{v} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

[5 marks]

QUESTION 4

In the case of the vectors $\boldsymbol{x} = \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}$ and $\boldsymbol{u} = \dfrac{1}{3} \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} \in \mathbb{R}^3$ find the constant $\alpha \in \mathbb{R}$ and the vector $\boldsymbol{x}^{\perp} \in \mathbb{R}^3$ that is **perpendicular** to the (unit) vector $\boldsymbol{u}$ such that

$$\boldsymbol{x} = \alpha\, \boldsymbol{u} + \boldsymbol{x}^{\perp}$$

[5 marks]

QUESTION 5

Consider the following vectors in $\mathbb{R}^3$:

$$\boldsymbol{u}_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad \boldsymbol{u}_2 = \frac{1}{\sqrt{6}}\begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix} \quad \text{and} \quad \boldsymbol{x} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}.$$

Given that $\boldsymbol{u}_1$, $\boldsymbol{u}_2$ are **orthonormal** (with respect to the usual inner product on $\mathbb{R}^3$) find the constants $\alpha_1, \alpha_2 \in \mathbb{R}$ and the vector $\boldsymbol{x}^\perp \in \mathbb{R}^3$, that is **perpendicular** to $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$, such that

$$\boldsymbol{x} = \alpha_1\, \boldsymbol{u}_1 + \alpha_2\, \boldsymbol{u}_2 + \boldsymbol{x}^\perp.$$

[10 marks]

QUESTION 6

Let $\wp$ be the plane through the origin which is perpendicular to the (unit) vector $\boldsymbol{\xi} = \frac{1}{5}\begin{bmatrix} -3 \\ 0 \\ 4 \end{bmatrix}$. When light from infinity, shining parallel to $\boldsymbol{\xi}$, falls on the parallelogram that is spanned by the vectors:

$$\boldsymbol{u} = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} \quad \text{and} \quad \boldsymbol{v} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

it casts a shadow on the plane $\wp$, find the area of this shadow.

[10 marks]

QUESTION 7: The curve $\boldsymbol{\mathcal{C}}$ in $\mathbb{R}^3$ that is parametrized by $\gamma : \mathbb{R} \to \mathbb{R}^3 : t \mapsto \begin{bmatrix} t^2 + t \\ t^2 \\ t^2 - t \end{bmatrix}$ intersects the plane $x - y + z - 4 = 0$. Determine **all** points of intersection.

[5 marks]

QUESTION 8: If $\gamma(t)$ is the position at time t of a particle that is moving at constant speed, s, in a circle that is centred at the origin and is of radius r show that the acceleration vector, $\ddot{\gamma}(t)$, is given by

$$\ddot{\gamma}(t) = -\left(\frac{s}{r}\right)^2 \gamma(t)$$

Hint: At each time t write $\ddot{\gamma}(t)$ in the form $\ddot{\gamma}(t) = a(t)\gamma(t) + b(t)\dot{\gamma}(t)$ where $a(t),\, b(t) \in \mathbb{R}$. Now use the facts that $\langle\, \gamma(t),\, \gamma(t)\,\rangle = r^2$ and $\langle\, \dot{\gamma}(t),\, \dot{\gamma}(t)\,\rangle = s^2$ to determine $a(t)$ and $b(t)$.

[5 marks]