Interlude 2: A crash course in Linear Algebra.

→ In all that follows we consider the vector space $\mathbb{R}^2$ (the plane) or $\mathbb{R}^3$ (the space).

→ We see vectors as coordinates:

$$\mathbb{R}^2 = \mathbb{R} \times \mathbb{R} = \{ (x,y),\ x \in \mathbb{R},\ y \in \mathbb{R} \},$$

$$\mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R} = \{ (x,y,z),\ x \in \mathbb{R},\ y \in \mathbb{R},\ z \in \mathbb{R} \}.$$

→ We will write $\mathbb{R}^n$ or $\mathbb{R}^m$, meaning $n = 2$ or 3 or 1. and $m = 2$ or 3 or 1

→ The real line $\mathbb{R}$ is also to be considered so that $n = 1$ or $m = 1$ is also a case we consider.

→ If you fancy abstraction, you should know that all that we will say remains true for higher dimensional spaces.

$$\mathbb{R}^n = \underbrace{\mathbb{R} \times \cdots \times \mathbb{R}}_{n \text{ times}}.$$

→ If you like to stay grounded, remember n, m are $1, 2$ or 3.

DEFINITION: A linear mapping $f : \mathbb{R}^n \to \mathbb{R}^m$ is a mapping such that for every $\vec{u}, \vec{v} \in \mathbb{R}^n$, for every $\lambda, \mu \in \mathbb{R}$,

$$f(\lambda \vec{u} + \mu \vec{v}) = \lambda f(\vec{u}) + \mu f(\vec{v}).$$

REMARK: When $n = m = 1$ these are the <u>linear functions</u>

$$f : \mathbb{R} \to \mathbb{R}$$
$$x \mapsto ax \qquad \text{for some } a \in \mathbb{R}.$$

DEFINITION: a m by n matrix A is an array of numbers

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1} \\ a_{21} & \ddots & & \\ \vdots & & \ddots & \\ a_{m1} & & & a_{mn} \end{bmatrix}$$

m lines

n columns

PRODUCT OF a MATRIX AND A VECTOR:

For A a m by n matrix and $\vec{v} \in \mathbb{R}^n$, we can define

$A\vec{v} \in \mathbb{R}^m$, <u>the product of A and $\vec{v}$</u>,

which we explain now on examples.

THEOREM: To every linear mapping

$$f : \mathbb{R}^n \to \mathbb{R}^m$$

corresponds a m by n matrix A

(NOTICE THAT m and n are permuted),

and for every $\vec{u} \in \mathbb{R}^n$,

$$f(\vec{u}) = A\vec{u}.$$