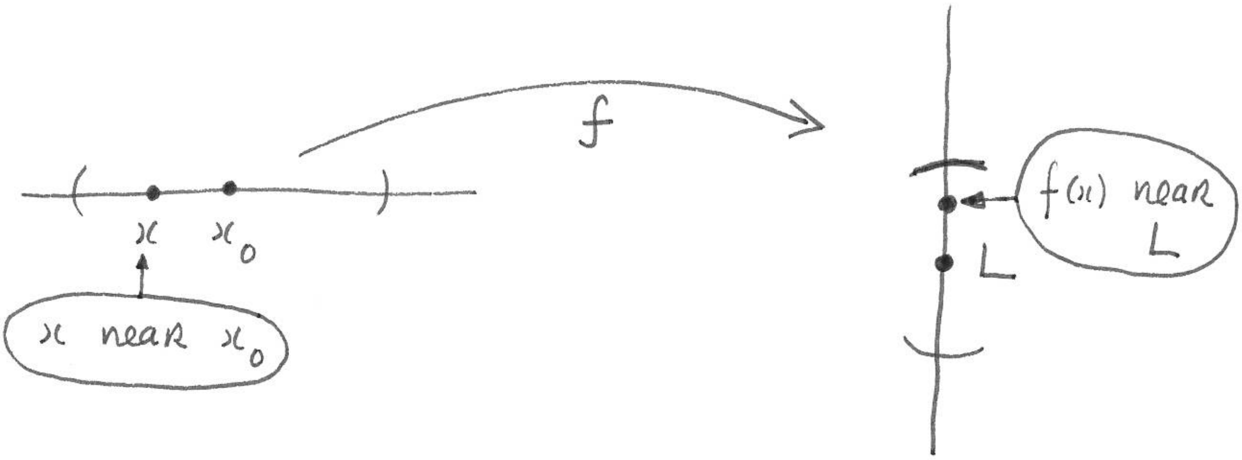
MS 221 Homework Set 1

Q1

(a) "In words" $\lim_{x \to x_0} f(x) = L$ means

$$\boxed{x \text{ near } x_0} \implies \boxed{f(x) \text{ is near } L}$$

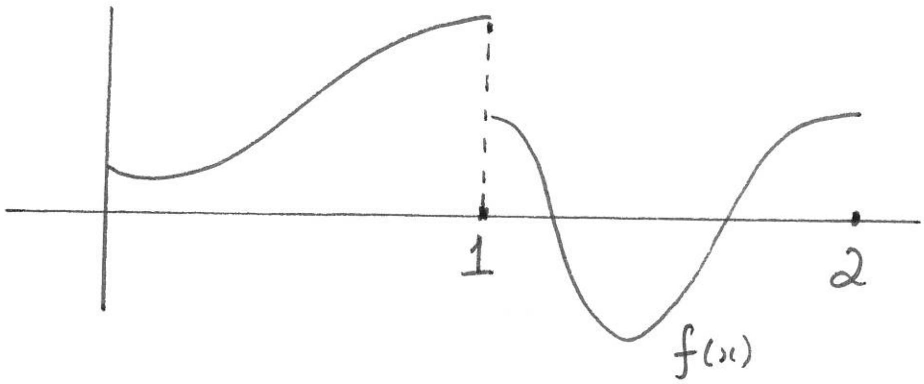
(b)



Q2

$$\boxed{\begin{array}{c} f \text{ is continuous} \\ \text{at } x_0 \end{array}} \overset{\text{def}^n}{\iff} \boxed{\lim_{x \to x_0} f(x) = f(x_0)}$$

Q3



Q4 For f to be continuous at $x = 2$ it is necessary and sufficient that

$$f(2) = \lim_{x \to 2} f(x).$$

In particular, this limit must exist. In this case,

$$\lim_{x \to 2} f(x) = \lim_{x \to 2} \frac{x^2 + 3x - 10}{x - 2}$$

$$= \lim_{x \to 2} \frac{(x-2)(x+5)}{(x-2)}$$

$$= \lim_{x \to 2} (x+5) = 7$$

So f will be continuous at $x = 2$ if and only if we define $f(2) = 7$.

Q5

$$\frac{d}{dx}\ln(x^2+\cos x) = \left(\frac{d}{du}\ln u\right)\frac{du}{dx}$$

Chain Rule

$$= \frac{1}{u}\cdot(2x-\sin x)$$

$$= \frac{2x-\sin x}{x^2+\cos x}$$

$$\frac{d}{dx}e^{\sin 4x} = \left(\frac{d}{du}e^{u}\right)\cdot\frac{du}{dx}$$

Chain Rule

$$= e^{u}\cdot 4\cos 4x = e^{\sin 4x}\cdot 4\cos 4x$$

Q6

$$\frac{d}{dx}\tan^{-1}\left(\frac{a}{x}\right) = \left(\frac{d}{du}\tan^{-1}u\right)\frac{du}{dx}$$

Chain Rule

$$= \frac{1}{1+u^2}\cdot\left(-\frac{a}{x^2}\right)$$

$$= \frac{1}{1+\left(\frac{a}{x}\right)^2}\cdot\left(\frac{-a}{x^2}\right) = \frac{-a}{x^2+a^2}$$

Q7 Here we are using the two equivalent versions of THE FUNDAMENTAL THEOREM OF CALCULUS (as given in Q8 & Q9)

$$\int_a^x \frac{d}{dt}\left(\frac{1}{1+t^{20}}\right)dt = \left(\frac{1}{1+x^{20}}\right) - \left(\frac{1}{1+a^{20}}\right)$$

$$\frac{d}{dx}\int_a^x \frac{1}{1+t^{20}}\,dt = \left(\frac{1}{1+x^{20}}\right)$$

Q8 This is usually regarded as the second version of The Fundamental Theorem of Calculus. That is:

$$\int_a^x \frac{df}{dt}(t)\,dt = f(x) - f(a)$$

Q9 This is usually regarded as the first version of The Fundamental Theorem of Calculus. That is;

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x).$$