MS 221 — Homework Set (1)

(Review of First Year Calculus)

QUESTION 1

In the case of a function $f : (a,\ b) \to \boldsymbol{R} : x \mapsto f(x)$ do the following, where $x_0 \in (a,\ b)$ and $L \in \boldsymbol{R}$:

(a) State in your own words what is meant by: $\lim_{x \to x_0} f(x) = L$.

(b) Draw a picture to illustrate the content of the above statements.

QUESTION 2

In the case of a function $f : (a,\ b) \to \boldsymbol{R} : x \mapsto f(x)$ and a point $x_0 \in (a,\ b)$ explain what is meant by the statement:

$$f \text{ is continuous at } x_0.$$

QUESTION 3

Sketch the graph of any function $f : (0,\ 2) \to \boldsymbol{R} : x \mapsto f(x)$ which is continuous everywhere **except at the point** $x = 1$.

QUESTION 4

Is it possible to assign a value to $f(2)$ in such a way that the function

$$f : \boldsymbol{R} \to \boldsymbol{R} : x \mapsto \begin{cases} \dfrac{x^2 + 3x - 10}{x - 2} & \text{when } x \neq 2 \\ f(2) & \text{when } x = 2 \end{cases}$$

is continuous at $x = 2$. Justify your answer.

QUESTION 5

Calculate the following

$$\frac{d}{dx}\ln(x^2+\cos x) \qquad \text{and} \qquad \frac{d}{dx}e^{\sin 4x}$$

QUESTION 6

Given that $\dfrac{d}{du}\tan^{-1}u = \dfrac{1}{1+u^2}$ calculate $\dfrac{d}{dx}\tan^{-1}\left(\dfrac{a}{x}\right)$ where a is constant.

QUESTION 7

Evaluate the following

$$\int_a^x \frac{d}{dt}\left(\frac{1}{1+t^{20}}\right) dt \qquad \text{and} \qquad \frac{d}{dx}\int_a^x \frac{1}{1+t^{20}}\, dt$$

QUESTION 8

If a function $f : [a,\ b] \to \boldsymbol{R} : t \mapsto f(t)$ has continuous derivative, determine a function $A(x)$ for which the identity

$$\int_a^x \frac{df}{dt}(t)\, dt \;=\; A(x)$$

is valid for every $x \in (a,\ b)$.

QUESTION 9

If a function $f : [a,\ b] \to \boldsymbol{R} : t \mapsto f(t)$ is continuous, determine a function $B(x)$ for which the identity

$$\frac{d}{dx}\int_a^x f(t)\, dt \;=\; B(x)$$

is valid for every $x \in (a,\ b)$. Draw a picture to illustrate why continuity of f is essential.