MS 221 — Homework Set (2)

(The Inner Product and The Cross Product)

QUESTION 1

In the case of the vectors $\boldsymbol{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ and $\boldsymbol{v}_2 = \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix} \in \boldsymbol{R}^3$ calculate $\|\boldsymbol{v}_1\|$, $\|\boldsymbol{v}_2\|$, $\langle \boldsymbol{v}_1,\, \boldsymbol{v}_2 \rangle$ and hence determine the angle between $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$.

QUESTION 2

In the case of the vectors $\boldsymbol{x} = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$ and $\boldsymbol{u} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \in \boldsymbol{R}^3$ find the constant $\alpha \in \boldsymbol{R}$ and the vector $\boldsymbol{x}^{\perp} \in \boldsymbol{R}^3$ which is **perpendicular** to the (unit) vector $\boldsymbol{u}$ such that

$$\boldsymbol{x} = \alpha\, \boldsymbol{u} + \boldsymbol{x}^{\perp}$$

QUESTION 3

Consider the following vectors in $\boldsymbol{R}^3$:

$$\boldsymbol{u}_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad \boldsymbol{u}_2 = \frac{1}{\sqrt{3}} \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}, \quad \boldsymbol{u}_3 = \frac{1}{\sqrt{6}} \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix} \quad \text{and} \quad \boldsymbol{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$

Now show that $\boldsymbol{u}_1$, $\boldsymbol{u}_2$, $\boldsymbol{u}_3$ are **orthonormal** (with respect to the usual inner product in $\boldsymbol{R}^3$) and hence express the vector $\boldsymbol{v}$ as a linear combination of them.

QUESTION 4

Find an equation of the plane in $\boldsymbol{R}^3$ which passes through the point $\boldsymbol{p} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and is perpendicular to the vector $\boldsymbol{n} = \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}$.

QUESTION 5

Find the perpendicular distance from the plane $2x - y + z = 12$ to the origin.

QUESTION 6

Find the cross product of the vectors in $\boldsymbol{R}^3$

$$\boldsymbol{v}_1 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \quad \boldsymbol{v}_2 = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$$

and hence determine the equation of the plane in $\boldsymbol{R}^3$ which passes through the origin, $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$.

QUESTION 7

Find the equation of the plane in $\boldsymbol{R}^3$ which passes through the points

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

QUESTION 8

Find the area of the parallelogram which is spanned by the vectors

$$\boldsymbol{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \text{and} \quad \boldsymbol{u} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \in \boldsymbol{R}^3$$

QUESTION 9

Let $\boldsymbol{\mathcal{P}}$ be the plane through the origin which is perpendicular to the (unit) vector $\boldsymbol{\xi} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$. When light from infinity, shining parallel to $\boldsymbol{\xi}$, falls on the parallelogram described in question 8 it casts a shadow on the plane $\boldsymbol{\mathcal{P}}$. Find the area of this shadow.