# MS 221 — Homework Set (3)

## (Parametrization of Curves in $\boldsymbol{R}^2$ and $\boldsymbol{R}^3$)

**QUESTION 1**

Parametrize the line joining the points $\boldsymbol{p} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ and $\boldsymbol{q} = \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix} \in \boldsymbol{R}^3$.

**QUESTION 2**

Parametrize the line in $\boldsymbol{R}^3$ which is determined by the intersection of the planes

$$\begin{array}{ccccccc} x & + & 2y & - & z & = & 2 \\ 3x & + & 5y & + & z & = & 1 \end{array}$$

**QUESTION 3**

Parametrize the circle on the $xy$-plane which has centre $(3, -5)$ and radius $= 2$.

**QUESTION 4**

Parametrize the **ellipse** on the $xy$-plane which is determined by the equation:

$$\frac{(x-1)^2}{5^2} + \frac{(y-3)^2}{7^2} = 1.$$

**QUESTION 5**

Parametrize the **hyperbola** on the $xy$-plane which is determined by the equation:

$$\frac{(x-1)^2}{5^2} - \frac{(y-3)^2}{7^2} = 1.$$

**QUESTION 6**

Parametrize the **parabola** on the $xy$-plane which is determined by the equation:

$$(y+1)^2 = 12(x-5).$$

**QUESTION 7**

Find $\lim_{t\to 1} \gamma(t)$ where $\gamma : \boldsymbol{R} \setminus \{1\} \to \boldsymbol{R}^2 : t \mapsto \begin{bmatrix} \dfrac{t^3 - t}{t-1} \\ \\ \dfrac{\sin(t-1)}{t-1} \end{bmatrix}$.

## QUESTION 8

If $\boldsymbol{C}$ is the curve in $\boldsymbol{R}^3$ which is parametrized by

$$\gamma : \boldsymbol{R} \to \boldsymbol{R}^3 : t \mapsto \begin{bmatrix} t^3 - t \\ (3t+5)^2 \\ t^2 + 1 \end{bmatrix}$$

do the following:

(a) Calculate $\gamma(-1)$.

(b) Calculate $\dfrac{d\gamma}{dt}(t)$ when $t = -1$.

(c) Parametrize the tangent **LINE** to the curve $\boldsymbol{C}$ at the point $\gamma(-1)$.

## QUESTION 9

Let $\gamma(t)$ be as given in Question 8. If the **position** vector of a particle at time $t$ is $\gamma(t)$ find the **velocity** and **acceleration** vectors of this particle at time $t$.

## QUESTION 10

Fix an origin $\mathbf{0}$ (in 3-dimensional space) and let $\boldsymbol{r}(t)$ be the position vector (relative to $\mathbf{0}$) of a particle $p$ at time $t$. We define:

(a) $\boldsymbol{v}(t) := \dfrac{d\boldsymbol{r}}{dt}(t)$ the velocity vector of $p$ at time $t$.

(b) $\boldsymbol{M} := m\boldsymbol{v}$ the momentum vector, note ($m =$ mass of $p$)

(c) $\boldsymbol{F} :=$ the force on $p$.

(d) $\boldsymbol{A_q} := \boldsymbol{x_q}(t) \times \boldsymbol{M}$ called the angular momentum of the particle $p$ about the fixed point $\boldsymbol{q}$. The vector $\boldsymbol{x_q}(t) := \boldsymbol{r}(t) - \boldsymbol{q}$ is the position vector of the particle $p$ **relative to point** $\boldsymbol{q}$ at time $t$.

(e) Torque about $\boldsymbol{q} := \boldsymbol{x_q}(t) \times \boldsymbol{F}$.

**Newton's Second Law** states:

$$\boldsymbol{F} = \frac{d\boldsymbol{M}}{dt}$$

and **The Principle of Angular Momentum** states:

$$\frac{d\boldsymbol{A_q}}{dt} = \boldsymbol{x_q} \times \boldsymbol{F} \quad \text{for every point } \boldsymbol{q} \text{ in 3-space.}$$

Show that these laws are equivalent.