MS 221 — Homework Set (8)

(Integration Along Curves)

QUESTION 1

If the curve $\mathcal{C}$ in $\boldsymbol{R}^3$ is **parametrized** by $\gamma : [0,\, 1] \to \boldsymbol{R}^3 : t \mapsto \begin{bmatrix} 1 \\ 2t \\ 3t^2 \end{bmatrix}$, calculate the line integral $\displaystyle\int_{\mathcal{C}} xy\, dz$

QUESTION 2

If $\mathcal{C}$ is the straight line in $\boldsymbol{R}^3$ which joins the point $\boldsymbol{p} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ to the point $\boldsymbol{q} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ calculate the line integral $\displaystyle\int_{\mathcal{C}} xy\, dz$.

QUESTION 3

Evaluate the line integral $\displaystyle\int_{\mathcal{C}} 3x\, dx - y^2\, dy + dz$ where the curve $\mathcal{C}$ in $\boldsymbol{R}^3$ is **parametrized** by

$$\gamma : [1,\, 3] \to \boldsymbol{R}^3 : t \mapsto \begin{bmatrix} 2t \\ 1 - t \\ 2 + t^2 \end{bmatrix},$$

QUESTION 4

If the **vector field** $\boldsymbol{F} : \boldsymbol{R}^3 \to \boldsymbol{R}^3$ and the curve $\mathcal{C}$ in $\boldsymbol{R}^3$, which is **parametrized** by $\gamma : [-1,\, 1] \to \boldsymbol{R}^3 : t \mapsto \gamma(t)$, are determined by the formulae

$$\boldsymbol{F}(x,\, y,\, z) = \begin{bmatrix} x - y^2 \\ -z \\ xyz^2 \end{bmatrix} \qquad \text{and} \qquad \gamma(t) = \begin{bmatrix} t^2 \\ 2 - t \\ 4 \end{bmatrix}$$

find the **work done by $\boldsymbol{F}$ over the curve $\mathcal{C}$**.

QUESTION 5

If the **vector field** $\boldsymbol{F} : \boldsymbol{R}^3 \to \boldsymbol{R}^3$ and the curve $\boldsymbol{\mathcal{C}}$ in $\boldsymbol{R}^3$, which is **parametrized** by $\gamma : [-3,\, 3] \to \boldsymbol{R}^3 : x \mapsto \gamma(x)$, are determined by the formulae

$$\boldsymbol{F}(x,\, y,\, z) = \begin{bmatrix} xy \\ -yz \\ zx \end{bmatrix} \qquad \text{and} \qquad \gamma(x) = \begin{bmatrix} x \\ \sqrt{9 - x^2} \\ 2 \end{bmatrix}$$

find the **work done by $\boldsymbol{F}$ over the curve $\boldsymbol{\mathcal{C}}$**.

Hint: You might consider reparametrizing the curve $\boldsymbol{\mathcal{C}}$.

QUESTION 6

Consider the scalar field $\varphi : \boldsymbol{R}^3 \to \boldsymbol{R} : (x,\, y,\, z) \mapsto \varphi(x,\, y,\, z) := x(y^2 + z^2)$ and let $\boldsymbol{F} : \boldsymbol{R}^3 \to \boldsymbol{R}^3$ be the vector field $\boldsymbol{F} = \nabla\varphi$. If the curve $\boldsymbol{\mathcal{C}}$ in $\boldsymbol{R}^3$ is parametrized by

$$\gamma : [0,\, 1] \to \boldsymbol{R}^3 : t \mapsto \begin{bmatrix} \cos \pi t^2 \\ \left(\dfrac{1}{1 + \sin^2 \pi t}\right) \\ t \end{bmatrix}$$

find the **work done by $\boldsymbol{F}$ over the curve $\boldsymbol{\mathcal{C}}$**.

Hint: THINK before you rush off to calculate a line integral!

QUESTION 7

Evaluate the **iterated integral** $\displaystyle\int_0^1 \int_{y^2}^{y} 2xy \; dx \, dy.$

QUESTION 8

Sketch the region $\Omega := \{(x,\, y) \in \boldsymbol{R}^2\}$ which is determined by the inequalities:

$$x - 2 \;\le\; y \;\le\; 2x - x^2$$

and hence write $\displaystyle\int\!\!\int_\Omega f(x,\, y)\, dA$ as an **iterated integral**.

QUESTION 9

Sketch the region $\Omega := \{(x,\, y) \in \boldsymbol{R}^2\}$ which is determined by the inequalities:

$$\frac{1}{x} \;\le\; y \;\le\; 1 - \frac{1}{x}$$

and hence **evaluate** $\displaystyle\int\!\!\int_\Omega \frac{2\,y}{x^2}\, dA.$