MS321 Tutorial 1

1. Consider the family of mappings

$$M = \{f_k : \mathbb{R} \to \mathbb{R} : x \mapsto x + k \mid k \in \mathbb{Z}\}$$

Show that composition on M is a commutative operation. Is there a particular k for which every element of M is of the form $(f_k)^l$ for some $l \in \mathbb{Z}$? Is there more than one such k?

2. Consider the finite set D_4 of mappings from $\mathbb{R}^2$ to itself given by the 8 matrices

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix},$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}.$$

Form the multiplication table for D_4 under the operation of composition of mappings or multiplication of the corresponding matrices.

3. Suppose $f : A \to B$ and $g : B \to C$ are both one-to-one. Show that the composition $g \circ f : A \to C$ is also one-to-one.

4. Suppose $f : A \to B$ and $g : B \to C$ are both onto. Show that the composition $g \circ f : A \to C$ is also onto.

5. Compute the powers of the permutations given by

$$f : (1, 2, 3, 4, 5) \mapsto (2, 3, 4, 5, 1)$$

and

$$g : (1, 2, 3, 4, 5) \mapsto (2, 3, 1, 5, 4).$$

The order of a permutation h is the smallest positive integer k with $h^k = \mathrm{id}_A$. What are the orders of f and g in S_5?

MS321 Tutorial 1 hints

1. Composition in M is just composition of real valued function. Compute $f_3 \circ f_2(x)$. Which f_k is this? Compute $f_l \circ f_m(x)$. Which f_k is this? Compute $(f_3)^2$, $(f_3)^5$, $(f_3)^{-3}$. Is M equal to the set of all $(f_3)^k$ or are there some missing?

2. The first matrix is the identity, the second, third, fifth and eight are reflections in lines, the fourth, sixth and seventh are rotations.

3. Start with the assumption that $(g \circ f)(a_1) = (g \circ f)(a_2)$ and prove $a_1 = a_2$. You should just need the definitions of one-to-one and of composition.

4. Start with an element c in C and find an element a in A with $(g \circ f)(a) = c$. You should just need the definitions of onto and of composition.

5. The order is how many times you need to apply the permutation before you get the identity. Compute the orders of the two given permutations.