**MS321 Tutorial 3**

1. (Division Algorithm) If $n$ is a four digit number with digits $abcd$ and $m = 10$, what are the values of $q$ and $r$ in the Division Algorithm?

2. If $a$ and $b$ are positive integers define their least common multiple, written $\text{lcm}(a, b)$, to be the integer $l$ satisfying
(i) $l$ is a common multiple of $a$ and $b$ and
(ii) if $m$ is any common multiple of $a$ and $b$ then $l \leq m$.
Prove that $ld = ab$ where $d = \gcd(a, b)$.

3. Express the gcd of 270 and 336 as an integral linear combination of 270 and 336.

4. If $a$ and $b$ are integers with gcd $d$ satisfying $a \mid c$ and $b \mid c$ show that $ab \mid cd$.

5. If $a$ and $b$ are coprime integers and $a \mid bc$ show that $a \mid c$.

6. Show that there is (a) really only one group with 2 elements and (b) only one group with 3 elements. Hint: Use group multiplication tables and the cancellation laws.

7. Suppose $G$ is a group satisfying $g^2 = e$ for every element $g$ in $G$. Prove that $G$ is abelian.

**MS321 Tutorial 3 Hints**

1. Try some four digit numbers like 3275 and 5802. Compare $q$ and $r$ with the original numbers.

2. Write $a$ and $b$ as multiples of $d$. Show that the multiples are coprime. Now $ab$ is this product of four numbers, two of which are $d$. So dropping one of the $d$'s should give a number $l$ satisfying $ld = ab$. Show that this $l$ satisfies (i) and (ii). Expect to use the fact that the multiples above are coprime.

3. As in lecture.

4. The 'magic' equation is $d = xa + yb$.

5. The 'magic' equation is now $1 = xa + yb$.

6. In a group with 2 elements, the elements are $e$ (the identity) and $a$. You will need to use $a \neq e$. In a group with 3 elements, the elements are $e$ (the identity), $a$ and $b$. You will need to use $a \neq e$, $b \neq e$ and $a \neq b$. Try to work out what $ab$ is using cancellation to eliminate two of the three possibilities.

7. If $a$ and $b$ are two elements in such a $G$ then $a^2 = e$ and $b^2 = e$. However we also know $(ab)^2 = e$.