MS321 Algebra, tutorial 4

1. The symmetry group of the cube in $\mathbb{R}^3$ with vertices $(\pm 1, \pm 1, \pm 1)$, consists of the forty eight 3×3 matrices, with the property that each row and column contains exactly one non-zero entry which is $+1$ or -1. There is a tetrahedron with vertices $(1,1,1)$, $(1,-1,-1)$, $(-1,1,-1)$ and $(-1,-1,1)$, sitting inside the cube. Which of the cube's symmetries preserve this tetrahedron?

2. In $GL(2, \mathbb{R})$, the group of invertible 2×2 real matrices, describe all the elements of the form g^k for $k \in \mathbb{Z}$ and $g = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$.

3. Prove that the intersection of two subgroups of a group is also a subgroup. If m and n are positive integers which cyclic subgroup of $\mathbb{Z}$ is $m\mathbb{Z} \cap n\mathbb{Z}$?

4. How many generators does a cyclic group of order 11 have? How many generators does a cyclic group of order 60 have? If p is a prime, how many generators does a cyclic group of order p have? If p and q are distinct primes, how many generators does a cyclic group of order pq have?