MS341 Algebra, tutorial 5

1. List the left cosets of $\langle(2,2)\rangle$ in $\mathbf{Z}_6 \times \mathbf{Z}_{10}$.

2. For $H < G$ show that $gH = Hg$ for every $g \in G$ if and only if $ghg^{-1} \in H$ for every $g \in G$ and for every $h \in H$.

3. Use Lagrange's Theorem to find all the subgroups of D_4, the symmetry group of the square. Hint: There are 10.

4. Use Lagrange's Theorem to prove that if p is a prime number and n is any integer then $n^p - n$ is a multiple of p. Hint: Look at cases $p \,|\, n$ and $p \not|\, n$ using Q1 from Tutorial 4 for second case.

MS341 Algebra, tutorial 5 hints

1. Big group should have 60 elements, subgroup should have 15, so there should be 4 cosets. The operation is addition, so gH will look like $g + H$.

2. You just need to understand the definitions of left and right cosets. For $gH = Hg$ show $gH \subseteq Hg$ and $Hg \subseteq gH$. Note that

$$gh_1 = h_2g \quad \Leftrightarrow \quad gh_1g^{-1} = h_2$$

3. This is hard. All Lagrange's Theorem says is that the orders are $1, 2, 4, 8$. Subgroups of size 1 or 8 are what you expect. Subgroups of size 2 are cyclic since 2 is prime. This should leave three subgroups of size 4.

4. Write n as $qp + r$ with $0 \leq r < p$. Question is now about r. The case $r = 0$ should be easier. If $1 < r < p$ then r is coprime to p.