MS321 Algebra, tutorial 7

1. Determine all the homomorphisms fro $\mathbf{Z}_{15}$ to $\mathbf{Z}_{18}$.

2. The group $\mathbb{Z}_{30}^*$ has 8 elements. There is a possibility it is isomorphic to one of $\mathbb{Z}_8$ or $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$. Show that is not isomorphic to either.

3. Recall that for $\sigma = (i_1, i_2, \ldots, i_k)$ a k-cycle in S_n and $\tau \in S_n$,

$$\tau \circ \sigma \circ \tau^{-1} = (\tau(i_1), \tau(i_2), \ldots, \tau(i_k)).$$

Apply this to the case $\sigma(i, i+1)$ and $\tau = (i+1, j)$ for $j > i+1$ to get a new proof of the result in Q1 of Tutorial 2.

4. Suppose G is a group and define the set

$$Z(G) = \{x \in G \mid xg = gx \text{ for all } \ g \in G\},$$

that is, the subset of G consisting of those elements which commute with all elements of G. Show that $Z(G)$ is a subgroup of G. Show that $Z(G)$ is normal in G.